**Academic Year 2021-2022**

**Exam 2 – Part II - Maximum duration: 3 hours**

**Problem 1 [2 points]**

|  |  |
| --- | --- |
| The figure shows a diagram of how an inverted pendulum works. The force applied to the carriage allows the pendulum rod (for simplicity, it is assumed that all the mass is at the end) to be kept around the vertical position defined by . he mass of the carriage is considered to be kg, the mass of the ball kg, m/s2 is the acceleration due to gravity, and the length of the pendulum m.  The linearised equation of the inverted pendulum around the vertical position is given by: |  |

It is assumed that an attempt will be made to control its position with a proportional gain controller .

1. Using the root locus technique, analyse the stability and qualitative behaviour of the closed-loop system for positive and negative values of . Explain the result obtained. **[0.5 points]**
2. Repeat the previous section, but in this case using the Nyquist stability criterion. **[0.5 points]**
3. Repeat the previous sections, but in this case using the Routh-Hurwitz stability criterion. **[0.5 points]**
4. For a value of , draw the Bode diagram on the next page and calculate the phase margin and gain margin of the closed-loop system, indicating them on the diagram. Explain whether it makes sense to apply these relative stability margins in this system. **[0.5 points]**

**Problem 2 [2 points]**

1. Taking into account what was done in section 4 of the previous exercise and for the same value of , consider whether it is possible to design a phase lead or phase lag network that achieves a phase margin of 25º for the system. Design the network if this is possible. Draw the root locus of the resulting system and, qualitatively (without calculating the inverse transform), the closed-loop step response, commenting appropriately in terms of transient and steady state. **[0.75 points]**
2. To control the system around the vertical , a zero reference is imposed, requiring the design of a system that rejects disturbances in the form of a unit step at the plant input, so that the pendulum returns to its vertical position, with dynamics dictated by a relative damping coefficient of 0.6 and a 2% settling time equal to 2.2 seconds (it is foreseeable that there may be some additional poles). Propose a controller that can control the system with these specifications and draw the closed-loop step response to the disturbance at the plant input. What would the output be with this controller if the disturbance entered at the plant output? **[1.25 points]**

**Problem 3 [1 point]**

For the inverted pendulum problem described by the linearised equation:

and considering regulation at the origin () and that only one disturbance acts at the plant input in the form of a unit step indicated in the previous problem.

1. Calculate an internal description of the system where the state variables are the angular position of the pendulum and its angular velocity (it is assumed that both can be measured using incremental encoders and tachometers, respectively). **[0.25 points]**
2. Maintaining the same disturbance rejection specification as in the previous section, propose a linear state vector feedback control system that can reject a step disturbance in . Draw a schematic diagram of the control system, calculate the matrices and vectors necessary for the controller design, and indicate the steps to be followed to solve the problem. **[0.75 points]**

**Imagen que contiene biombo, edificio, jaula

El contenido generado por IA puede ser incorrecto.**